UNIT-I INTRODUCTION TO FLUID STATICS

Module -1

Part-A: Distinction between a fluid and a solid - characteristics of fluids

1. Distinction between a Fluid and a Solid

1. All matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious by the physical observation but the technical distinction lies with the reaction of the two to an applied shear or tangential stress.
2. A solid can resist a shear stress by a static deflection; a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied.
3. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Fluids	Solids
Fluids have no shape	Solids have a definite shape
Fluids cannot sustain a shear force, i.e., a fluid is always in motion under shear force	Solids can sustain a shear force; i.e., they remain static under shear force
Stress is a function of the rate of strain; thus, a fluid had a `dynamic' state	Stress is a function of strain, thus a solid maintains a static or `quasi-static' state.
The static properties of a fluid cannot be extended to dynamic properties.	The static properties of a solid can be extended to dynamic properties.

2. Characteristics of fluids

2.1. Density: Density of a fluid is defined as the ratio of the mass of a fluid to its volume. It is represented by 'ρ (rho)'.

$$\rho = \frac{Mass\ of\ fluid}{Volume\ of\ the\ fluid}$$

Units: S.I. unit: kg/m^3; C.G.S. unit: g/cm^3

2.2. Specific weight: Specific weight of a fluid is the ratio between the weight of a fluid to its volume. It is denoted by 'w'.

$$w = \frac{Weight\ of\ fluid}{Volume\ of\ the\ fluid} = \frac{Mass\ of\ fluid \times Acceleration\ due\ to\ gravity}{Volume\ of\ the\ fluid}$$

$$w = \rho g$$

Units: S.I. unit: N/m^3

2.3. Specific volume: Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass (or)

volume per unit mass of a fluid is called specific volume.

$$Specific\ volume = \frac{Volume\ of\ fluid}{Mass\ of\ the\ fluid} = \frac{1}{\rho}$$

Units: S.I. unit: m^3/kg; C.G.S. unit: cm^3/g

It is commonly applied to gases.

2.4. Specific gravity: Specific gravity of a fluid is defined as the ratio of the density of a fluid to the density of a standard fluid. For liquids, the standard fluid is taken as water and for gases, the standard fluid is taken as air. It is represented by symbol 'S'

Mathematically,

$$S\ (for\ liquids) = \frac{Density\ of\ liquid}{Density\ of\ the\ water}$$

$$S\ (for\ gases) = \frac{Density\ of\ gas}{Density\ of\ the\ air}$$

Therefore, density of a liquid = $S \times$ density of water

Thus, if the specific gravity is known, then the density of the fluid can be calculated.

Example: Specific gravity of the mercury is 13.6.

Therefore, density of the mercury is $13.6 \times 1000 = 13600\ kg/m^3$

2.5. Viscosity: Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

(or)

Viscosity is a quantitative measure of a fluid's resistance to flow. More specifically, it determines the fluid strain rate that is generated by a given applied shear stress.

One can easily move through **air**, which has very low viscosity. Movement is more difficult in **water**, which has **50 times** higher viscosity. Still more resistance is found in **SAE 30 oil**, which is **300 times** more viscous than water. Try to slide your hand through **glycerin**, which is **five times** more viscous than SAE 30 oil, or **blackstrap molasses**, another factor of five higher than glycerin. Fluids may have a vast range of viscosities.

When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and $u + du$ as shown in the below Fig 1.1, the viscosity together with relative velocity causes a shear stress τ, acting between the fluid layers as in Fig. 1.2. The shear strain angle $\delta\theta$ will continuously grow with time as long as the stress τ is maintained, the upper surface moving at speed du larger than the lower surface. Common fluids such as water, oil and air show **a linear relation between shear and resulting strain rate**.

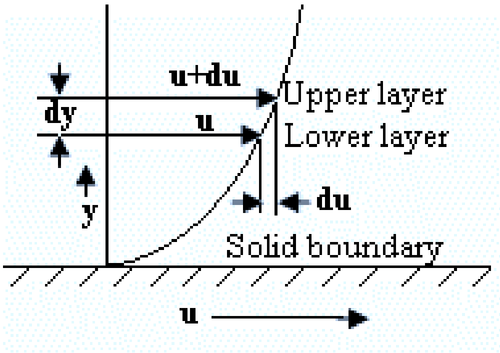


Fig. 1.1 Velocity variation near a solid boundary

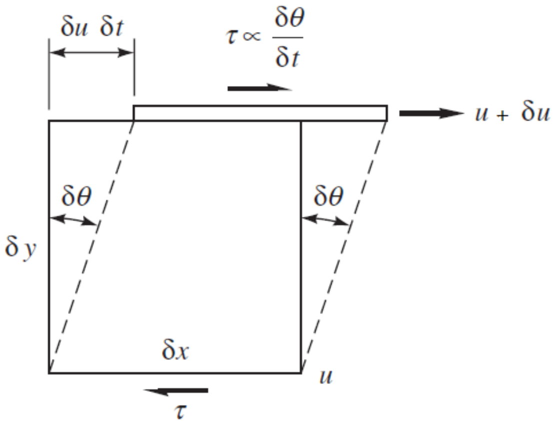


Fig. 1.2 Shear stress causes continuous shear deformation and a fluid element of thickness dy straining at a rate $\delta\theta/\delta t$

$$\tau \; \alpha \; \frac{\delta\theta}{\delta t}$$

$$\text{From Fig. 1.2., } \tan \delta\theta = \frac{\delta u \delta t}{\delta y}$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient.

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

From above equation, the applied shear is also proportional to the velocity gradient for the common linear fluids. *This is known as Newton's law of viscosity.* The constant of proportionality is the viscosity coefficient or Dynamic viscosity μ. Mathematically, it is expressed as

$$\boxed{\tau = \mu\frac{d\theta}{dt} = \mu\frac{du}{dy}}$$

Unit: Thus, $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$

S.I. unit of viscosity $= \frac{\boldsymbol{Newton-Sec}}{\boldsymbol{m^2}} = \frac{\boldsymbol{N-s}}{\boldsymbol{m^2}} = \boldsymbol{pa - s}$

M.K.S. unit of viscosity $= \frac{kg}{m-sec}$

C.G.S. Unit of viscosity **Poise** $= \frac{\boldsymbol{dyne-s}}{\boldsymbol{cm^2}} = \frac{\boldsymbol{gm}}{\boldsymbol{cm-sec}}$

Conversion of CGS unit to other units

$$\boxed{\begin{array}{c}\textbf{One poise} = \mathbf{0.1}\,\frac{\boldsymbol{N-s}}{\boldsymbol{m^2}} = \mathbf{0.1}\,\frac{\boldsymbol{kg}}{\boldsymbol{m-sec}} \\ \textbf{One centipoise} = \mathbf{10^{-2}}\,\boldsymbol{poise} = \mathbf{0.01}\,\boldsymbol{poise}\end{array}}$$

2.6. Kinematic viscosity: Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is represented by $'\nu'$.

$$\nu = \frac{Dynamic\ viscosity}{density} = \frac{\mu}{\rho}$$

Unit: S.I. & M.K.S. unit of kinematic viscosity $= \frac{\boldsymbol{m^2}}{\boldsymbol{s}}$

C.G.S. unit of kinematic viscosity $= \frac{\boldsymbol{cm^2}}{\boldsymbol{s}} = \boldsymbol{stoke}$

Conversion of CGS unit to other units

$$\boxed{\begin{array}{l}\mathbf{1}\,\boldsymbol{stoke} = \frac{\boldsymbol{cm^2}}{\boldsymbol{s}} = \mathbf{10^{-4}}\,\frac{\boldsymbol{m^2}}{\boldsymbol{s}} \\ \mathbf{1}\,\boldsymbol{centistoke} = \mathbf{0.01}\,\boldsymbol{stoke}\end{array}}$$

Variation of viscosity with temperature and pressure: viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. **Temperature** has a *strong effect* and **pressure** a *moderate effect* on viscosity.

The viscosity of gases and most liquids increases slowly with pressure.

Gases viscosity increases with temperature, The reason for this is, in gases the cohesive forces are small and molecular momentum predominates. With increase in temperature molecular momentum transfer increases and hence viscosity of gases increases.

Whereas *liquids viscosity decreases with temperature.* This is due to fact that, the cohesive forces predominate the molecular momentum transfer due to densely packed molecules in liquids and with the rise in temperature, the cohesive forces decrease with the result of decreasing viscosity.

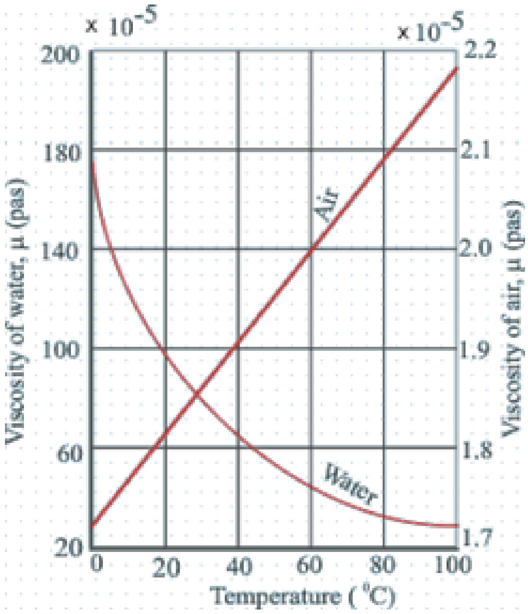


Fig. 1.3 Change of viscosity of water and air under 1 atm

No-slip Condition of Viscous Fluids

- It has been established through experimental observations that the relative velocity between the solid surface and the adjacent fluid particles is zero whenever a viscous fluid flows over a solid surface. This is known as **no-slip condition.**
- This behavior of no-slip at the solid surface is not same as the wetting of surfaces by the fluids. For example, mercury flowing in a stationary glass tube will not wet the surface, but will have zero velocity at the wall of the tube.
- **The wetting property results from surface tension, whereas the no-slip condition is a consequence of fluid viscosity**.

2.7. Compressibility: Compressibility of any substance is the measure of its change in volume under the action of external forces. The normal compressive stress on any fluid element at rest is known as *hydrostatic pressure* ‘p’ and arises as a result of innumerable molecular collisions in the entire fluid.

- *The degree of compressibility of a substance is characterized by the bulk modulus of elasticity (K), which is defined as the ratio of compressive stress to volumetric strain.*

$$K = \lim_{\nabla V \to 0} \left(\frac{-\Delta p}{\Delta V / V} \right)$$

$$\therefore Compressibility = \frac{1}{K} = \text{reciprocal of the bulk modulus}$$

Note:

For a given mass of a substance, the change in its volume and density satisfies the relation

$$\Delta m = 0 \Longrightarrow \Delta(\rho V) = 0$$

$$\Longrightarrow \frac{\Delta V}{V} = -\frac{\Delta \rho}{\rho}$$

Substituting above relation in bulk modulus of elasticity (K) Eq., we get

$$K = \lim_{\nabla V \to 0}\left(\frac{-\Delta p}{\Delta V/V}\right) = \lim_{\nabla \rho \to 0}\left(\frac{\Delta p}{\Delta \rho/\rho}\right) = \rho\frac{dp}{d\rho}$$

Values of K for liquids are very high as compared with those of gases. Therefore, liquids are usually termed as incompressible fluids though, in fact, no substance is theoretically incompressible with a value of E as ∞. For example, the bulk modulus of elasticity for water and air at atmospheric pressure are approximately $2 \times 10^6\ kN/m^2$ and $101\ kN/m^2$ respectively. It indicates that air is about 20,000 times more compressible than water. Hence water can be treated as incompressible.

2.8. Surface Tension: Surface tension is a binary property of the liquid and gas or two liquids which are in contact with each other and defines the interface.

The phenomenon of surface tension arises due to the two kinds of intermolecular forces

(i) Cohesion : The force of attraction between the molecules of a liquid by virtue of which they are bound to each other to remain as one assemblage of particles is known as the force of cohesion. This property enables the liquid to resist tensile stress.

(ii) Adhesion : The force of attraction between unlike molecules, i.e. between the molecules of different liquids or between the molecules of a liquid and those of a solid body when they are in contact with each other, is known as the force of adhesion. This force enables two different liquids to adhere to each other or a liquid to adhere to a solid body or surface.

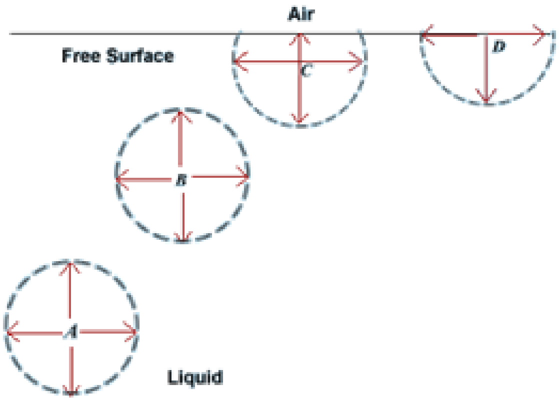


Fig. 1.4 The intermolecular cohesive force field in a bulk of liquid with a free surface

Observations made from Fig. 1.4

1. A and B experience equal force of cohesion in all directions, C experiences a net force interior of the liquid and that net force is maximum for D since it is at surface.
2. Work is done on each molecule arriving at surface against the action of an inward force. Thus, mechanical work is performed in creating a free surface or in increasing the area of the surface.
3. Therefore, a surface requires mechanical energy for its formation and the existence of a free surface implies the presence of stored mechanical energy known as free surface energy.
4. Any system tries to attain the condition of stable equilibrium with its potential energy as minimum. Thus, a quantity of liquid will adjust its shape until its surface area and consequently its free surface energy is a minimum.

*The magnitude of **surface tension** is defined as the tensile force acting on the surface of a liquid in contact with a gas (or on the surface between two immiscible liquids) such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by* $\boldsymbol{\sigma}$**.**

Unit: S.I. unit of surface tension = $\frac{N}{m}$

Note: If the interface is curved, a mechanical balance shows that there is a pressure difference across the interface, the pressure being higher on the concave side. i.e., *It is due to surface tension that a curved liquid interface in equilibrium results in a greater pressure at the concave side of the surface than that at its convex side.*

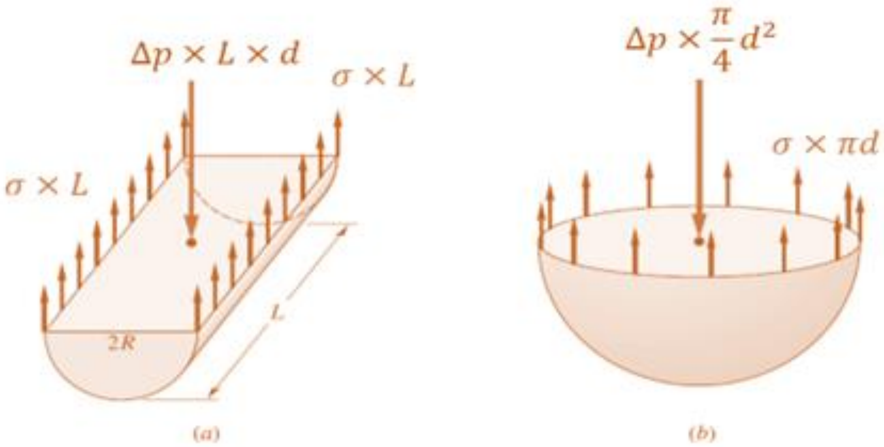


Fig. 1.5 Pressure change across a curved interface due to surface tension: (a) Interior of a liquid cylinder; (b) Interior of a spherical droplet

The pressure increase in the interior of a liquid cylinder is balanced by two -surface tension forces:

$$\Delta p \times L \times d = \sigma \times 2L$$

$$\Delta P = \frac{2\sigma}{d}$$

The pressure increase in the interior of a spherical droplet balances a ring of surface tension force:

$$\Delta p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\Delta p_{droplet} = \frac{4\sigma}{d}$$

The pressure rise inside a soap bubble, which has two interfaces with air, an inner and outer surface of nearly the same diameter, d:

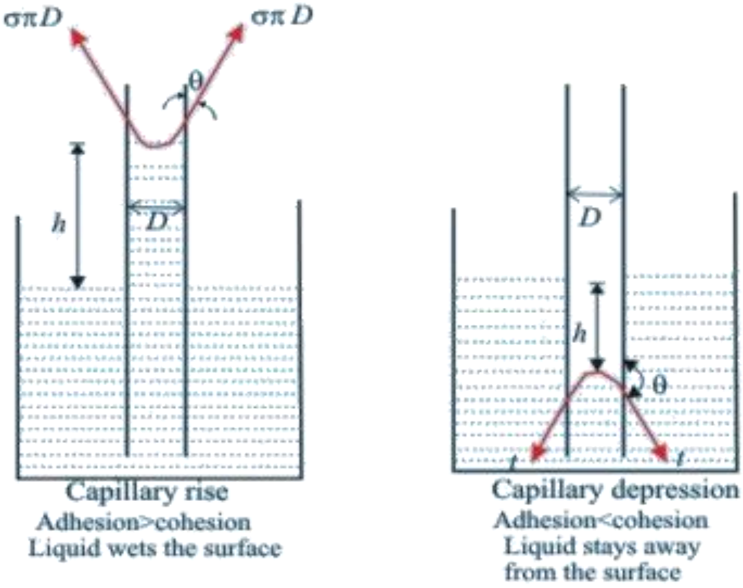
$$\Delta p_{bubble} \approx 2\Delta p_{droplet} = \frac{8\sigma}{d}$$

2.9. Capillarity: The interplay of the forces of cohesion and adhesion explains the phenomenon of capillarity.

- When a liquid is in contact with a solid, if the forces of adhesion between the molecules of the liquid and the solid are greater than the forces of cohesion among the liquid molecules themselves, the liquid molecules crowd towards the solid surface. The area of contact between the liquid and solid increases and **the liquid thus wets the solid surface.**

- The reverse phenomenon takes place when the force of cohesion is greater than the force of adhesion.

These adhesion and cohesion properties result in the phenomenon of capillarity by which a liquid either rises or falls in a tube dipped into the liquid depending upon whether the force of adhesion is more than that of cohesion or not.



Capillary rise
Adhesion>cohesion
Liquid wets the surface

Capillary depression
Adhesion<cohesion
Liquid stays away from the surface

Fig. 1.6 Phenomenon of capillarity

Note: 1. For pure water in contact with air in a clean glass tube, the capillary rise takes place with θ = 0°.

2. Mercury causes capillary depression with an angle of contact of about 130° in a clean glass in contact with air.

3. Since h varies inversely with D as found from Eq. $h = \frac{4\sigma cos\theta}{\rho g D}$; an appreciable capillary rise or depression is observed in tubes of small diameter only.

2.10. Vapour Pressure and Cavitation: All liquids have a tendency to evaporate when exposed to a gaseous atmosphere. The rate of evaporation depends upon the molecular energy of the liquid which in turn depends upon the type of liquid and its temperature.

The vapour molecules exert a partial pressure in the space above the liquid, known as vapour pressure.

If the space above the liquid is confined and the liquid is maintained at constant temperature, after sufficient time, the confined space above the liquid will contain vapour molecules to the extent that some of them will be forced to enter the liquid.

Eventually an equilibrium condition will evolve when the rate at which the number of vapour molecules striking back the liquid surface and condensing is just equal to the rate at which they leave from the surface.

The space above the liquid then becomes saturated with vapour. The vapour pressure of a given liquid is a function of temperature only and is equal to the saturation pressure for boiling corresponding to that temperature.

Hence, the vapour pressure increases with the increase in temperature. *Therefore, the phenomenon of boiling of a liquid is closely related to the vapour pressure.* In fact, when the vapour pressure of a liquid becomes equal to the total pressure impressed on its surface, the liquid starts boiling. *Thus, vapor pressure is the pressure at which a liquid-boils and is in equilibrium with its own vapor.* This concludes that boiling can be achieved either by raising the temperature of the liquid, so that its vapour pressure is elevated to the ambient pressure, or by lowering the pressure of the ambience (surrounding gas) to the liquid's vapour pressure at the existing temperature.

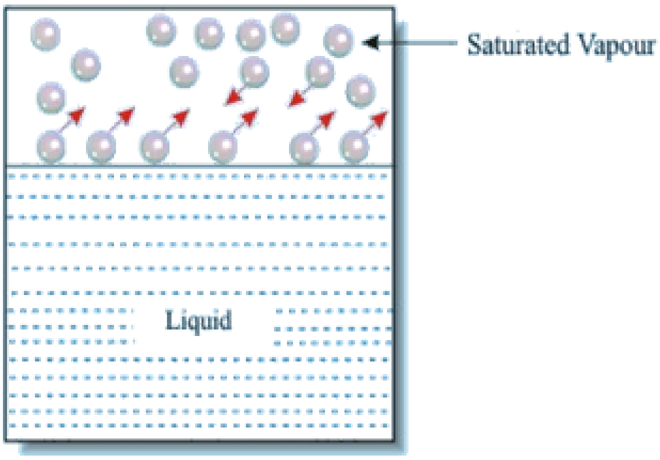


Fig. 1.7 To and fro movement of liquid molecules from an interface in a confined space as a closed surrounding

Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal or less than the vapour pressure, the vaporization of the liquid starts. The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as ***Cavitation.*** Thus, the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure.

Properties of standard liquid (water)

S. No	Property	Value
1.	Density at 4°C	$1000\ kg/m^3$
2.	Specific weight at 4°C	$9810\ N/m^3$
3.	Specific gravity at 4°C & 1 atm	1
4.	Dynamic viscosity at 20°C	$0.01\ poise$
5.	Kinematic viscosity at 20°C	$1\ centi\ stoke\ (cSt)$
6.	Surface tension of water in contact with air at 20°C	$0.073\ N/m$
7.	For pure water in contact with air in clean glass tube, capillary rise takes place with	$\theta = 0°$
8.	Mercury causes capillary depression with an angle of contact of about	$\theta = 130°$
9.	Bulk modulus of elasticity of water	$2.07\ GPa$
10.	Compressibility	$3.3 \times 10^{-6}\ (psi)^{-1}$

3. Types of fluids

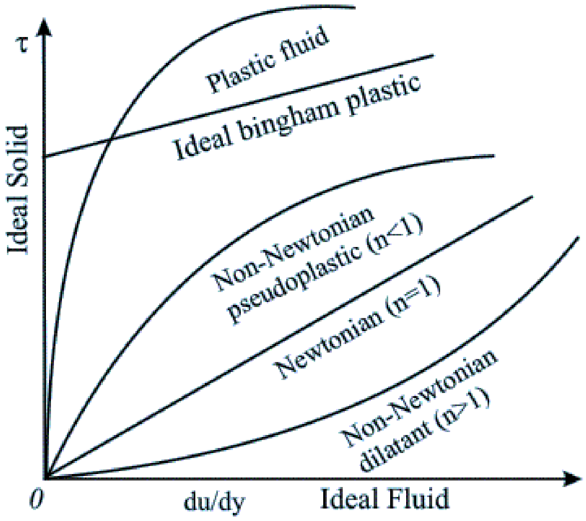


Fig. 1.8 Shear stress and deformation rate relationship of different fluids

The abscissa (x-axis) in Fig. 1.8 represents the behaviour of ideal fluids since for the ideal fluids the resistance to shearing deformation rate is always zero, and hence they exhibit zero shear stress under any condition of flow. The ordinate (y-axis) represents the ideal solid for there is no deformation rate under any loading condition.

Ideal fluid: Consider a hypothetical fluid having a zero viscosity ($\mu = 0$). Such a fluid is called an ideal fluid and the resulting motion is called as ideal or inviscid flow. In an ideal flow, there is no existence of shear force because of vanishing viscosity.

$$\tau = \mu \frac{du}{dy}; \;\; \text{Since } \mu = 0$$

All the fluids in reality have viscosity ($\mu > 0$) and hence they are termed as real fluid and their motion is known as viscous flow. Under certain situations of very high velocity flow of viscous fluids, an accurate analysis of flow field away from a solid surface can be made from the ideal flow theory.

3.1. Newtonian fluids: Fluids that follow the linear law $\tau = \mu \frac{du}{dy}$ are called Newtonian fluids. (or) A Newtonian fluid's viscosity remains constant, no matter the amount of shear applied for a constant temperature. These fluids have a linear relationship between viscosity and shear stress. *e.g.*, water, mineral oil, gasoline, alcohol, mercury etc.,

3.2. Nonnewtonian fluids: Fluids that do not follow the linear law $\tau = \mu \frac{du}{dy}$ are called nonnewtonian and are covered in rheology, Examples for these fluids are paints, different polymer solution, blood as do not obey the typical linear relationship, of τ and du/dy. In nonnewtonian fluids viscosity itself may be a function of deformation rate. For the nonlinear curves in Fig. 1.8., the slope at any point is called the apparent viscosity.

The non-Newtonian fluids are further classified as pseudo-plastic, dilatant and Bingham plastic.

Many mathematical models are available to describe the nonlinear "shear-stress vs deformation-rate" relationship of non-Newtonian fluids. But no general model can describe the constitutive equation ("shear stress vs rate of deformation" relationship) of all kinds of non-Newtonian fluids.

However, the mathematical model for describing the mechanistic behaviour of a variety of commonly used non-Newtonian fluids is the Power-Law model which is also known as Ostwald-de Waele model. According to Ostwald-de Waele model,

$$\tau = m \left|\frac{du}{dy}\right|^{n-1} \frac{du}{dy}$$

where m indicates flow consistency index and n denotes flow behaviour index

Hence viscosity for the Power-law fluids (as per Power Law model) can be described as

$$\mu = m \left|\frac{du}{dy}\right|^{n-1}$$

It can be observed from above that the viscosity of non-Newtonian fluid is function of deformation rate and **is often termed as apparent or effective viscosity**.

When $n = 1$, m equals μ, the model identically satisfies Newtonian model as a special case. When $n < 1$, the model is valid for **pseudoplastic fluids**, such as gelatine, blood, milk etc. When $n > 1$, the model is valid for **dilatant fluids**, such as sugar in water, aqueous suspension of rice starch etc.

There are some substances which require a *yield stress* for the deformation rate (i.e., the flow) to be established, and hence their constitutive equations do not pass through the origin thus violating the basic definition of a fluid. They are termed as **Bingham plastic**. For an ideal Bingham plastic, the shear stress- deformation rate relationship is linear.

Part-B: Fluid Pressure: Pressure at a point, Pascal's law, pressure variation with temperature, density and altitude. Piezometer, U-Tube Manometer, Single Column Manometer, U Tube Differential Manometer, pressure gauges.

1. Forces on Fluid Elements

Fluid element can be defined as an infinitesimal region of the fluid continuum in isolation from its surroundings.

Two types of forces exist on fluid elements

- **Body Force:** distributed over the entire mass or volume of the element. It is usually expressed per unit mass of the element or medium upon which the forces act. *Example: Gravitational Force, Electromagnetic force fields etc.*
- **Surface Force:** Forces exerted on the fluid element by its surroundings through direct contact at the surface. Surface force has two components:
  - *Normal Force:* along the normal to the area
  - *Shear Force:* along the plane of the area.

The ratios of these forces and the elemental area in the limit of the area tending to zero are called the normal and shear stresses respectively.

The shear force is zero for any fluid element at rest and hence the only surface force on a fluid element is the normal component.

2. Fluid pressure at a point

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA.

Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the *intensity of pressure* or simple *pressure (p).*

Mathematically, $p = \frac{dF}{dA}$

If the force (F) is uniformly distributed over the area (A), then the pressure at any point is given by $p = \frac{Force}{Area} = \frac{F}{A}$

$$\therefore F = p \times A$$

Units: S.I. unit: $\frac{Newton}{Sq.mt.} = \frac{N}{m^2} = Pascal$

Other commonly used units of pressure: $1kPa = 1000 \ {}^{N}/_{m^2}$

$$1bar = 100kPa = 10^5 \, {}^{N}/_{m^2}$$

2.1. Pascal's law of hydrostatics

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. (or)

The external static pressure applied on a confined liquid is distributed or transmitted evenly throughout the liquid in all directions.

Derivation of Pascal's Law

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in the Fig. 1.9, Let the width of the element is unity and p_x, p_y and p_z are the pressures acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are:

1. Pressure forces normal to the surfaces.
2. Weight of element in the vertical direction.

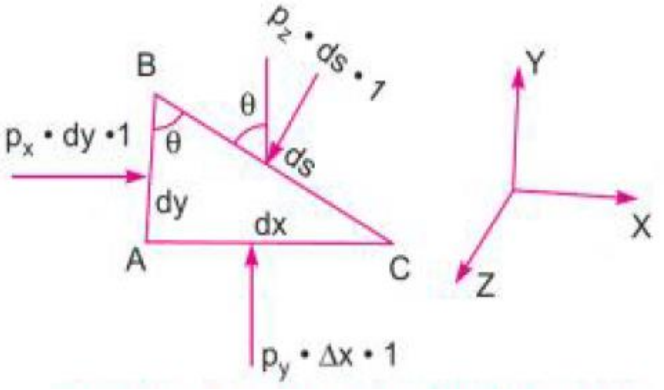


Fig. 1.9 Forces on fluid element

The forces on the faces are:

Force on the face AB = $p_x \times Area\ of\ face\ AB = p_x \times dy \times 1$

Similarly, force on the face AC = $p_y \times dx \times 1$

Force on the face BC = $p_z \times ds \times 1$

Weight of element = $\frac{AB \times AC}{2} \times 1 \times w$

where w = weight density of fluid

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p_z \times ds \times 1\, sin(90° - \theta) = 0$$

$$\therefore p_x \times dy \times 1 - p_z \times ds \times 1\, cos(\theta) = 0$$

But from Fig., $ds\, cos\theta = AB = dy$

$$\therefore p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

$$\therefore p_x = p_z \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Similarly resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1\, cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times w = 0$$

$$p_y \times dx - p_z \times ds\, sin\theta - \frac{dx \times dy}{2} \times w = 0$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$$\therefore p_y \times dx - p_z \times dx = 0$$

$$\therefore\ p_y = p_z \dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

From Eqs. (1) and (2), we have $p_x = p_y = p_z$

i.e., the choice of fluid element is completely arbitrary, which means the pressure at any point is the same in all directions.

Applications of Pascal's Law

Hydraulic Lift: Fig. 1.10. denotes a hydraulic lift. It works based on the principle of equal pressure transmission throughout a fluid (Pascal's Law).

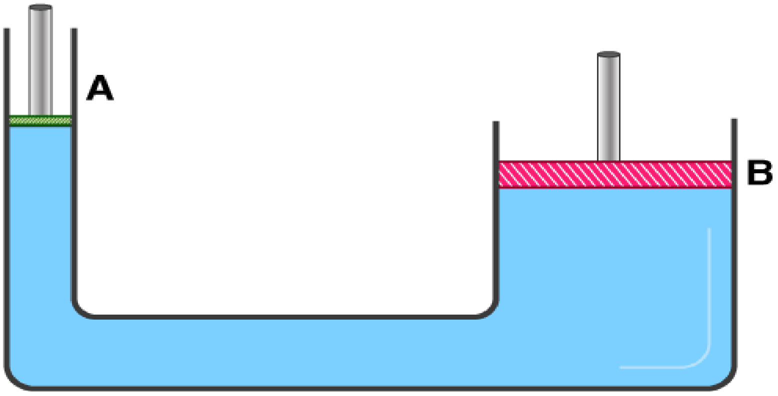


Fig. 1.10 Hydraulic lift

The construction is such that a narrow cylinder (in this case A) is connected to a wider cylinder (in this case B). They are fitted with airtight pistons on either end. The inside of the cylinders is filled with fluid that cannot be compressed.

Pressure applied at piston A is transmitted equally to piston B without diminishing the use of the fluid that cannot be compressed. Thus, piston B effectively serves as a platform to lift heavy objects like big machines or vehicles. A few more applications include a hydraulic jack and hydraulic press, and forced amplification is used in the braking system of most cars.

2.2. Pressure variation in a fluid at rest

The pressure at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate increase of pressure in a vertically downward direction must be equal to specific weight of the fluid at that point. This is proved as:

Consider a small fluid element as indicated in Fig. 1.11

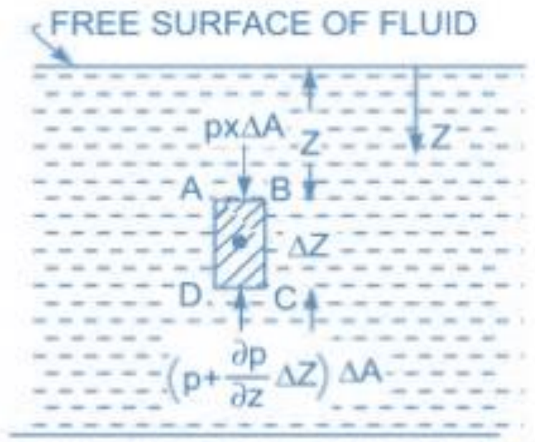


Fig. 1.11 Forces on a fluid element

Let $\Delta A =$ cross-sectional area of element

$\Delta Z =$ Height of fluid element

$p =$pressure on face AB

$Z =$Distance of fluid element from free surface

The forces acting on the fluid element are:

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.
2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial z}\Delta z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.
3. Weight of fluid element $= weight\ density \times volume = w \times (\Delta A \times \Delta z)$
4. Pressure forces on surfaces BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p \times \Delta A - \left(p + \frac{\partial p}{\partial z}\Delta z\right) \times \Delta A + w \times (\Delta A \times \Delta z) = 0$$

$$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial z}\Delta z \Delta A + w \times (\Delta A \times \Delta z) = 0$$

$$-\frac{\partial p}{\partial z}\Delta z \Delta A + w \times (\Delta A \times \Delta z) = 0$$

$$\frac{\partial p}{\partial z}\Delta z\Delta A = w \times (\Delta A \times \Delta z) \Rightarrow \frac{\partial p}{\partial z} = w$$

$$\therefore \frac{\partial p}{\partial z} = \rho g$$

The above equation states that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is known as Hydrostatic law.

By integrating the above equation for liquids, we get

$$\int dp = \int \rho g \, dz$$

$$p = \rho g z$$

Where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces. From the above equation we have $Z = \frac{p}{\rho g}$

Here Z is called Pressure Head

2.3. Units and scales of Pressure Measurement

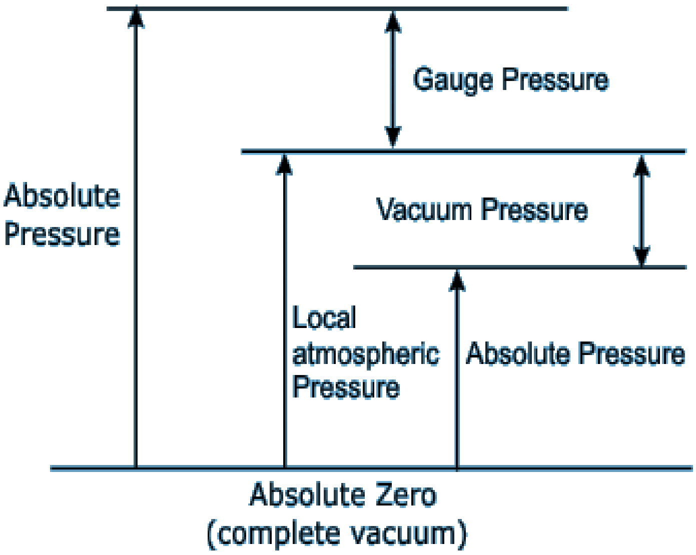


Fig. 1.12 The scale of pressure

Pascal (N/m^2) is the unit of pressure.

Pressure is usually expressed with reference to either absolute zero pressure (a complete vacuum) or local atmospheric pressure.

- **The absolute pressure:** It is the difference between the value of the pressure and the absolute zero pressure or it is measured with reference to absolute vacuum pressure.

$$p_{abs} = p - 0 = p$$

- **Gauge pressure**: It is the difference between the value of the pressure and the local atmospheric pressure (p_{atm}) or It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which atmospheric pressure is taken as datum. i.e., atmospheric pressure on the scale is marked as zero.

$$p_{gauge} = p_{abs} - p_{atm}$$

- **Vacuum Pressure**: If $p_{abs} < p_{atm}$ then the gauge pressure (p_{gauge}) becomes negative and is called the vacuum pressure. But one should always remember that hydrostatic pressure is always compressive in nature.

At sea-level and at 15°C, the international standard atmospheric pressure has been chosen as $p_{atm} = 101.32\ kN/m^2$

2.4. Pressure Measurement

The pressure of a fluid is measured by Manometers and Mechanical gauges

2.4.1. Manometers: Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. or A device that uses the surface area and weight of a liquid column to measure and indicate pressure is known as a manometer.

They are classified as simple manometers and differential manometers

(i) Simple manometer: It consists of a glass tube having one of its ends connected to a point where pressure is to be measured and the other end remains open to atmosphere. Common types of simple manometers are: piezometer, U-tube manometer and single column manometer.

a) Piezometer Tube The direct proportional relation between gauge pressure and the height h for a fluid of constant density enables the pressure to be simply visualized in terms of the vertical height, $h = \frac{p}{\rho g}$. The height h is termed as pressure head equivalent to pressure p.

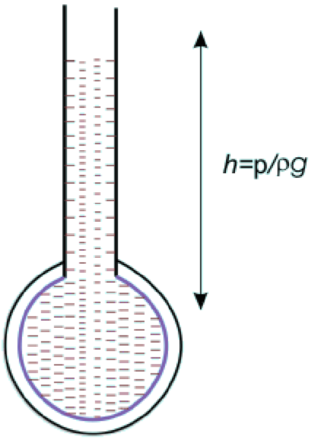


Fig. 1.13 A Piezometer tube

Piezometer is the simplest form of manometer used for measuring gauge pressures. If a small tube of sufficient length and open to atmosphere is connected to the pipe. Such a tube is called a *piezometer tube* as indicated in Fig. 1.13 and the height *h* is the measure of the gauge pressure of the fluid in the pipe. If such a piezometer tube of sufficient length were closed at the top and the space above the liquid surface were a perfect vacuum, the height of the column would then correspond to the absolute pressure of the liquid at the base. *This principle is used in the well-known mercury barometer to determine the local atmospheric pressure.*

b) U-tube manometer: It consists of glass tube bent in U-shape, one end of which is connected to a pipe or a container having a fluid (A) whose pressure is to be measured while the other end is open to atmosphere as shown in the Figs. 1.14 and 1.15. The lower part of the U-tube contains a liquid immiscible with the fluid A and is of greater density than that of A. This fluid is called the manometric fluid.

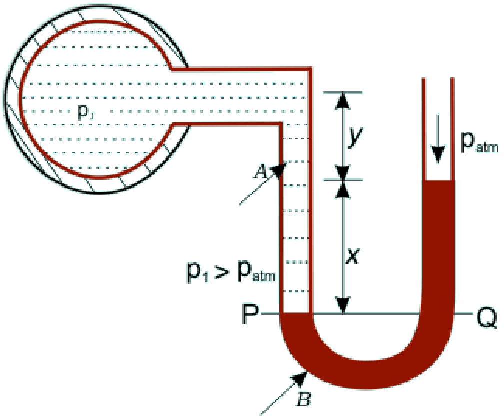


Fig. 1.14 For gauge pressure

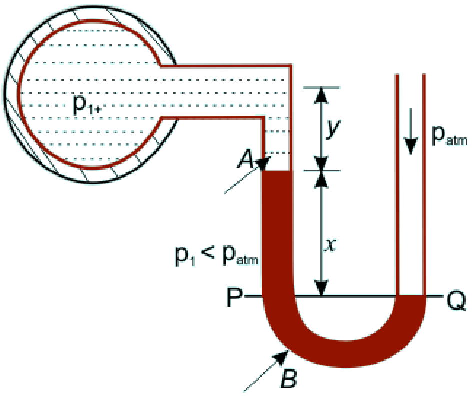


Fig. 1.15 For vacuum pressure

The pressures at two points *P* and *Q* (Fig. 1.14) in a horizontal plane within the continuous expanse of same fluid (the liquid B in this case) must be equal. Then equating the pressures at *P* and *Q* in terms of the heights of the fluids above those points, with the aid of the fundamental equation of hydrostatics, we have

$$p_1 + \rho_A g(y + x) = p_{atm} + \rho_B gx$$

$$\text{Hence, } p_1 - p_{atm} = (\rho_B - \rho_A)gx - \rho_A gy$$

Where p_1 is the absolute pressure of the fluid A in the pipe or container at its centre line, and p_{atm} is the local atmospheric pressure.

When the pressure of the fluid in the container is lower than the atmospheric pressure, the liquid levels in the manometer would be adjusted as shown in Fig. 1.15. Hence it becomes,

$$p_1 + \rho_A gy + \rho_B gx = p_{atm}$$

$$\text{Hence, } p_{atm} - p_1 = (\rho_A y + \rho_B x)g$$

c) Single column manometer: It is a modified form of a U-tube manometer in which a reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs of the manometer as shown in Figs. 1.16 & 1.17. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. Most common types of this type are: vertical single column manometer and Inclined single column manometer.

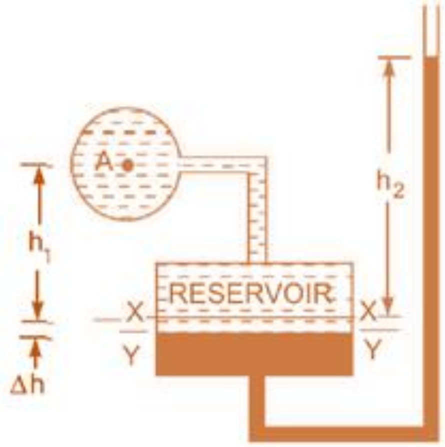


Fig. 1.16 Vertical single column manometer

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A}$$

Now consider the datum line $Y - Y$ as shown in Fig. 1.16. Then pressure in the right limb above $Y - Y = p_2 \times g \times (\Delta h + h_2)$

Pressure in the left limb above $Y - Y = p_1 \times g \times (\Delta h + h_1) + P_A$

Equating these pressures, we have

$$p_2 \times g \times (\Delta h + h_2) = p_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g(\Delta h + h_2) - \rho_1 g(\Delta h + h_1) = \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$P_A = \frac{a \times h_2}{A}[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected. Then $P_A = h_2\rho_2 g - h_1\rho_1 g$

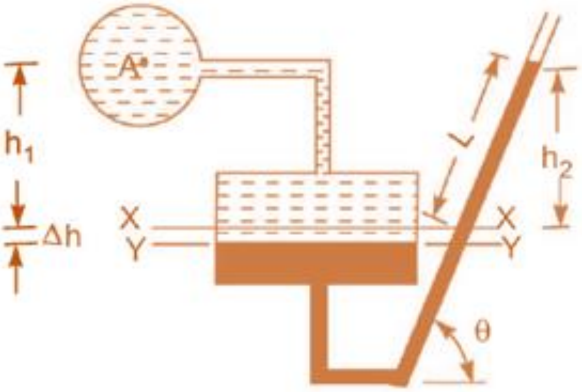


Fig. 1.17 Inclined single column manometer

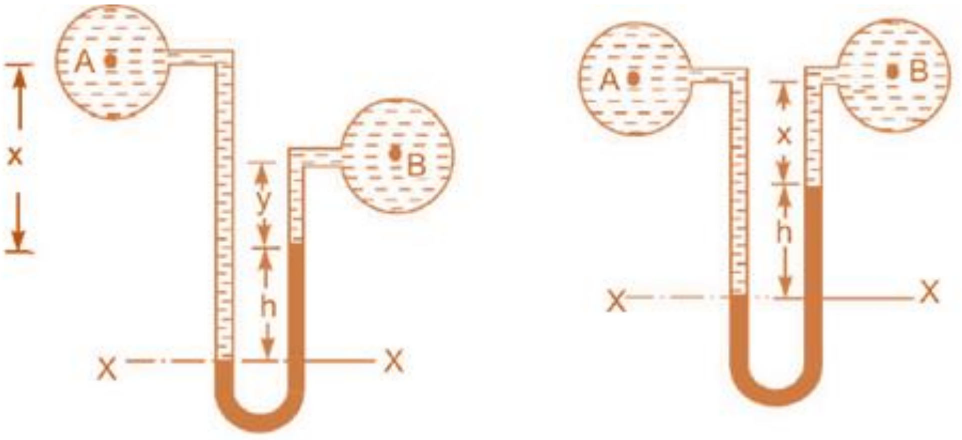
Fig. 1.17 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

$$P_A = h_2\rho_2 g - h_1\rho_1 g$$

But From Fig. 1.17, $h_2 = L\,sin\theta$

$$\therefore P_A = L\,sin\theta \times \rho_2 g - h_1\rho_1 g$$

(ii) Differential manometers: These are the devices used for measuring the difference of pressures between two points in the pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most common types of differential manometers are: U-tube differential manometer and Inverted U-tube differential manometer.



(a)Two pipes at different levels

(b) A and B are at the same level

Fig. 1.18 U-tube differential manometer

Let the two points A and B are at different level and also contains liquids of different specific gravity (Fig. 1.18a). These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B. Taking datum line at $X - X$.

Pressure above $X - X$ in the left limb $= \rho_1 g(h + x) + P_A$

Pressure above $X - X$ in the right limb$= \rho_g \times g \times h + \rho_2 \times g \times y + P_B$

Equating these two pressures, we have

$$\rho_1 g(h + x) + P_A = \rho_g \times g \times h + \rho_2 \times g \times y + P_B$$

$$\therefore P_A - P_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$\therefore P_A - P_B = hg\left(\rho_g - \rho_1\right) + \rho_2 g y - \rho_1 g x$$

If A and B are at the same level and contains the same liquid of density ρ_1 (Fig. 1.18b). Then the above equation reduces to

$$\therefore P_A - P_B = hg\left(\rho_g - \rho_1\right)$$

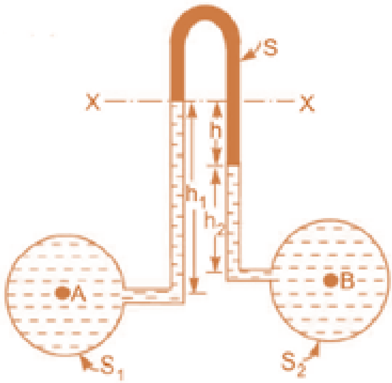


Fig. 1.19 Inverted U-tube differential manometer

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose pressure difference is to be measured. It is used for measuring difference of low pressures. Fig. 1.19 shows an inverted U-tube differential manometer connected to the two points $A\ and\ B$. Let the pressure at A is more than the pressure at B.

Taking $X - X$ as datum line. Then pressure in the left limb below $X - X$.

$$= P_A - \rho_1 \times g \times h_1$$

pressure in the right limb below $X - X$.

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating these two pressures, we have

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$\therefore P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

2.4.2. Pressure gauges:

The devices that are used for measuring pressure are called pressure gauges. Gauge pressure is the pressure relative to atmospheric pressure. For the pressures above atmospheric pressure, gauge pressure is positive. For the pressures below atmospheric pressure, gauge pressure is negative. The pressure gauge is also known as pressure meters or vacuum gauges.

Most gauges calculate the pressure relative to atmospheric pressure as the zero point. Hence, this form of reading is known as gauge pressure. Pressure gauges are analog as well as digital.

The pressure difference between the system and the atmosphere is given by the formula:

$$P = p_a + \rho g h$$

where, $P =$ pressure at any point & P_a = atmospheric pressure

From this, we can make out that the pressure at any point is always greater than the atmospheric pressure by $\rho g h$ amount.

when $P - P_a = \rho g h$

where, P =pressure of the system & P_a =atmospheric pressure

$(P - P_a)$ = pressure difference between the system and atmosphere.

$h \rho g$ = Gauge pressure

The difference between two pressures measured using the gauge is known as gauge pressure.

Types of Pressure gauges

i. Bourdon Tube Pressure Gauge

A bourdon tube is the most commonly used pressure gauge. It is a mechanical instrument that measures the pressure without an electric supply. It is made of steel to resist wear and corrosion. A bourdon tube pressure gauge can measure pressure from 0.6 to 7000 bar (8 to 10000 psi). It is compatible with liquid or gaseous media for vacuum, as well as low and high-pressure applications. It is a compact instrument that is ideal for heavy vibration application and dynamic pressure load. The bourdon tube pressure gauge is as shown in Fig. 1.20. They are radially formed tubes with an oval cross-section. The pressure of the measuring medium acts on the inside of the tube and produces a motion in the non-clamped end of the tube. This motion is the measure of the pressure and is indicated via the movement. The C-shaped Bourdon tubes, formed into an angle of approx. 250°, can be used for pressures up to 60 bar. For higher pressures, Bourdon tubes with several superimposed windings of the same angular diameter (helical tubes) or with a spiral coil in the one plane (spiral tubes) are used.

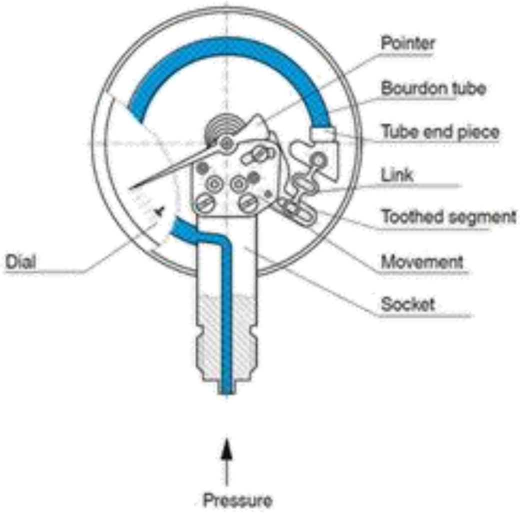


Fig. 1.20 Bourdon Tube Pressure Gauge

Applications of Bourdon Tube pressure gauge

They are used to measure medium to very high pressures.

Advantages of Bourdon tube pressure gauge

- These Bourdon tube pressure gauges give accurate results.
- Bourdon tube cost low.
- Bourdon tube are simple in construction.
- They can be modified to give electrical outputs.
- They are safe even for high pressure measurement.
- Accuracy is high especially at high pressures.

Limitations of bourdon tube pressure gauge

- They respond slowly to changes in pressure
- They are subjected to hysteresis.
- They are sensitive to shocks and vibrations.
- Amplification is a must as the displacement of the free end of the bourdon tube is low.
- It cannot be used for precision measurement.

ii. Diaphragm Pressure Gauge

It is the device used to measure the pressure of fluid in a system. It is purposefully designed to measure low-pressure intensities. A diaphragm pressure gauge is also known as a membrane pressure gauge.

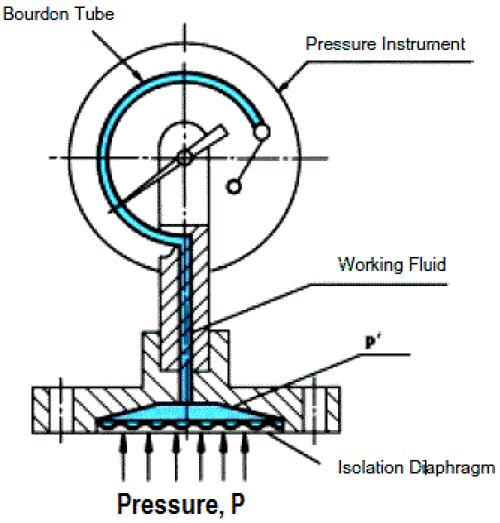


Fig. 1.21 Diaphragm Pressure Gauge

This device uses the deflection of a flexible thin membrane known as the diaphragm. The bottom side of the diaphragm is exposed to the pressure which is to be measured. Due to the applied pressure, the diaphragm deforms. That is the diaphragm tends to move upwards. This deformation of the diaphragm is proportional to the applied pressure. In a mechanical system, this deformation is magnified by the link – sector – pinion arrangement. That is, the linear displacement of the diaphragm is converted to a magnified rotary motion of the pinion. When the pinion rotates, it makes the pointer attached to it to assume a new position on the pressure calibrated scale which becomes a measure of the applied pressure. In case of the arrangement with the parallel plate capacitor, the movable plate moves upwards, thus reducing the gap between the plates. This makes the capacitance of the capacitor becomes a measure of the applied pressure. As the top side of the diaphragm is usually subjected to the atmospheric pressure, (generally less than the applied pressure) elastic diaphragm gauges usually read gauge pressure. Commonly used diaphragm materials are:

- Metals: stainless steel, Inconel, Monel, nickel and beryllium copper.
- Non – Metals: Nylon, Teflon and buna N rubber.

Applications of Elastic diaphragm gauges

- They are used to measure medium pressure.

- But they can also be used to measure low pressures including vacuum.
- They are used to measure draft in chimneys of boilers.

Advantages of Elastic diaphragm gauges

- Best advantage is they cost less
- They have a linear scale for a wide range
- They can withstand over pressure and hence they are safe to be used.
- No permanent zero shift.
- They can measure both absolute and gauge pressure, that is, differential pressure.

Limitations of Elastic diaphragm gauges

- Shocks and vibrations affect their performance and hence they are to be protected.
- When used for high pressure measurement, the diaphragm gets damaged.
- These gauges are difficult to be repaired.